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MARKET ANALYSIS WITH RATIONAL EXPECTATIONS: THEORY AND ESTIMATION

R. LaVar Huntzinger

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INTRODUCTION

Dynamic market analyses can be classified on the basis of their treatment of expected prices. The first, and until recently the accepted, approach has been to specify directly a process by which agents are supposed to form expectations. An alternative which has attracted increasing attention is to take expectations as the conditional expected values implied by the specification of the market process. To compare these approaches let's consider a market for which the government announces a sizable and previously unanticipated purchase to be made in the next period.

Structural equations for such a market can be developed describing optimal behavior conditional on beliefs about future prices. These equations come from assumptions about technology and agent's objectives. Fixing these aspects of the market process the structural equations hold for arbitrary values of their arguments. Since some types of behavior depend on beliefs about future prices, at least storage and also production if a lag is involved, the complete model must specify how relevant beliefs about future prices are formed.

Usually agents are assumed to form expectations according to some extrapolation rule. There are many variations of this approach, ranging from static expectations models in which the current price is expected to persist, to more sophisticated

adaptive and autoregressive schemes. In these approaches expected prices are equal to some combination of historical prices. Using this relation expected prices are eliminated from the structural equations by substitution. This gives a semi-reduced form which is used for parameter estimation and analysis. With this approach the semi-reduced form has the same invariance properties as the structural form.

An alternative approach is to take agent's expectations as the conditional expected values implied by economic theory. Such expectations were termed rational by Muth (1961). With this expectations hypothesis we cannot move so easily to a semi-reduced form from which expected prices are eliminated. Instead of using an a priori relation between expected and historical prices, the relevant structure must be specified and solved. Expected values for exogenous variables in future periods are relevant so processes implying them must be specified before the semi-reduced form can be obtained, and the semi-reduced form obtained is particular for that set of assumptions about exogenous structure.[1]

A disturbing feature of extrapolative treatments is that such models fail to express the economic content of the structural model. To see this consider again the market for which the government announced a purchase. Applying the structural

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[1]For examples see Lucas (1972), Sargent (1973), McCallum (1976), and Crawford (1975).

model to the next period will imply a prediction of some change in next period's price as a result of the announced purchase. At the same time the semi-reduced form implied by an extrapolative expectations rule treats the expectation of next period's price as some combination of historical prices which are unchanged by the announcement. When expectations are taken as extrapolations, we implicitly assume the participants persistently act on the basis of predictions which are correctibly different from those implied by the process [see Lucas and Prescott (1971) and Muth (1961)].

The rational expectations hypothesis (REH) avoids this inconsistency but has assumptions about the time paths of exogenous variables embedded in the semi-reduced form. Since changes in these time paths, such as would result from the government announcement, imply structural shifts impact analysis is slippery [see Lucas (1976)]. If such changes are frequent estimation of parameters is also a problem.

We would like an approach which provides the consistency of rational expectations while maintaining the familiar and desirable invariance properties of extrapolative models. This paper develops such an approach by maintaining separation of technology which implies the behavioral relations, and exogenous structure which determines the time paths of exogenous variables. This allows estimation of invariant equations and straightforward analysis of policy impacts while the exogenous structure changes. This framework also provides consistent estimators

for expected prices.

The OVERVIEW section, which follows, is a brief and suggestive statement of the approach giving its form without detail. The detailed development is given in the THEORY and ESTIMATION sections. The APPLICATION section reports a demonstration of the approach on the broiler market. Some extensions are suggested in the SUMMARY section.

I. OVERVIEW

Let's consider markets in which agents optimize expected values of quadratic objective functions subject to linear constraints. Certainty equivalence holds if uncertainty is limited to the coefficients in the linear parts of the objective functions and the righthand sides of the constraints [see Theil (1964)]. We maintain certainty equivalence so that the same decisions are indicated if stochastic components are replaced by the means of their predictive probability distributions.

Agent's behavior each period determines quantity variables such as consumption, production, and storage. With D_t representing the vector of such quantities, we represent aggregate behavior by

$$D_t = L_1(P_t, x_t, \varepsilon_t) \tag{1}$$

These are linear relations whose arguments are current and expected prices, P_t, an exogenous variable, x_t, and a vector of stochastic disturbance terms, ε_t.[2]

Quantities available in current and subsequent periods resulting from prior decisions determine the state of the market.

---

[2]Here and throughout the paper the exogenous variable can be thought of as representing a vector. The coefficient must then be interpreted as similarily dimensioned vector.

This state can be represented by a vector of such quantities, K_t. The evolution of this state satisfies a linear relation

$$K_{t+1} = AK_t + BD_t \tag{2}$$

In addition, market clearing implies linear relations between decision and state vectors for each period

$$GK_t = HD_t. \tag{3}$$

To illustrate, suppose production requires three periods and that consumption, d_t, production, q_t, and storage, s_t, decisions are made each period. Then the state and decision vectors are

$$K_t = \begin{bmatrix} s_{t-1} \\ q_{t-1} \\ q_{t-2} \\ q_{t-3} \end{bmatrix} \quad \text{and} \quad D_t = \begin{bmatrix} d_t \\ q_t \\ s_t \end{bmatrix}$$

The law of motion for the state vector could be

$$K_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} K_t + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} D_t$$

and market clearing condition

$$[1 \quad 0 \quad 0 \quad 1]\, K_t = [1 \quad 0 \quad 1]\, D_t .$$

Rational expectations imply a fixed point P_t with elements

$$p_t, \overset{*}{p}_{t+1}, \overset{*}{p}_{t+2}, \overset{*}{p}_{t+3} \cdots$$

where * indicates a mathematical expected value conditioned on information available in period t; e.g.,

$$\overset{*}{p}_{t+j} = \mathcal{E}(p_{t+j} | \mathcal{I}_t)$$

For given expected exogenous values, the fixed point is a price sequence for which the quantities implied by the behavioral relations satisfy the law of motion and market clearing conditions in all periods. This provides the internal consistency missing in extrapolative approaches.

Substituting from the behavioral relations into the market clearing conditions for the current and future periods gives linear conditions in prices, current and expected, exogenous values, current and expected, and the current state vector. The coefficients in these conditions are known functions of the structural parameters. These conditions are solved for prices, the fixed point, in terms of the other arguments, giving

$$p_t = L_2(X_t, K_t, \varepsilon_t) \tag{4}$$

where X_t is a vector with elements

$$x_t, \overset{*}{x}_{t+1}, \overset{*}{x}_{t+2}, \overset{*}{x}_{t+3} \cdots$$

Given these prices the behavioral relations determine decision quantities.

This provides a framework for analysis with the same invariance properties as extrapolative models. As long as the forms of agents' decision problems don't change analysis is straightforward. Changes in exogenous structure, such as government policy, imply changes in exogenous values, current and/or expected. As described above these imply changes in prices which imply changes in behavior.

This structure poses an unusual estimation problem. Conventional simultaneous equation procedures cannot be used because expected prices are not observed. In addition, the simultaneous system includes identities containing the parameters to be estimated, and expected values of exogenous variables for all future periods. We describe an instrumental variable procedure which gives consistent estimators of the structural parameters. Since the coefficients in (4) are known functions of the structural parameters we also have consistent estimators of those coefficients. Then (4) provides consistent estimators for expected prices. This suggests the possibility of performing the usual two or three stage least-squares procedure using estimates of expected prices instead of observations. We show that the resulting estimator is also consistent. Iteration may provide estimators with greater asymptotic efficiency.

II. THEORY

In this section we derive a prototype market structure and develop the analysis in its context. From market clearing conditions and the behavioral relations come conditions which are solved for prices. In this framework we make a distinction between technology and exogenous structure. Assuming no changes in technology and knowledge of the structural parameters we describe how impacts of changes in exogenous structure can be analyzed.

Market Structure

The following assumptions provide a structure specific enough for definite results and flexible enough to represent a class of markets.

A2.1 Consumers are "passive," responding to the current price and some exogenous influences so that aggregate consumption, d_t, is linear in the price, p_t, an exogenous variable, x_t^1, and a stochastic disturbance term, ε_t^1, with expected value zero.

$$d_t = a_1 + b_1 p_t + c_1 x_t^1 + \varepsilon_t^1 \tag{5}$$

A2.2 Production comes from a large number of identical firms each small relative to the market. Production requires h periods with the quantity determined at the start of the

process. The average cost of producing an amount q in period t is

$$a_t^1(q)/q = \alpha_{10} + \tfrac{1}{2}\alpha_{11}q + \alpha_{12}x_t^2 \qquad (6)$$

with $\alpha_{11} > 0$ and x_t^2 an exogenous variable involved in production.

A2.3 Similarly a large number of identical firms store the good from period to period with each being small relative to the market. The average cost of storing a quantity s from period t to period t+1 is

$$a_t^2(s)/s = \alpha_{20} + \tfrac{1}{2}\alpha_{21}s + \alpha_{22}x_t^3 \qquad (7)$$

with $\alpha_{21} > 0$ and where x_t^3 is an exogenous variable affecting storage.

A2.4 Firms engaged in production and storage act as if maximizing the expected present value of their profits.

A2.5 In evaluating expected values all participants use the same notions of structure and have access to the same information which includes or implies:

- The values of the parameters in (5), (6), and (7),
- The values of exogenous variables in the current period,
- The expected values of exogenous variables in future periods,
- The values of consumption, production, and storage in part periods,

and • the price in the current period.

A2.6 All market activity is represented as consumption, production, or storage and the market clears each period.

Under these assumptions the representative producer chooses production quantities q_t, q_{t+1}, q_{t+2}, etc. which maximize

$$\gamma_t = \mathcal{E}\{\sum_{j=0}^{\infty} \rho^j (p_{t+h+j} - \alpha_{10} - \tfrac{1}{2}\alpha_{11} q_{t+j} - \alpha_{12} x^2_{t+j}) q_{t+j} | \phi_t\}$$

where $1 < \rho < 0$. This is a quadratic programming problem with a unique maximum which satisfies the conditions for certainty equivalence. The first-order conditions are necessary and sufficient. They are

$$\rho^j [\overset{*}{p}_{t+h+h} - \alpha_{10} - \alpha_{11} q_{t+j} - \alpha_{12} \overset{*2}{x}_{t+j}] = 0$$

for $j \geq 0$. If the quantity actually produced differs from that planned by a stochastic component, ε^2_t having expected value zero, then aggregate production satisfies

$$q_t = a_2 + b_2 \overset{*}{p}_{t+h} + c_2 x^2_t + \varepsilon^2_t \tag{8}$$

with $a_2 = \alpha_{10}/\alpha_{11}$, $b_2 = 1/\alpha_{11}$, and $c_2 = -\alpha_{12}/\alpha_{11}$. Expected production quantities for future periods satisfy

$$\overset{*}{q}_{t+j} = a_2 + b_2 \overset{*}{p}_{t+h+j} + c_2 \overset{*2}{x}_{t+j} \tag{8a}$$

The representative firm engaged in storage chooses quantities s_t, s_{t+1}, s_{t+2}, etc. which maximize

$$\gamma_t = \mathcal{E}\{\sum^{\infty} \rho^j (\rho p_{t+j+1} - p_{t+j} - \alpha_{20} - \tfrac{1}{2}\alpha_{21}) s_{t+j} - \alpha_{22} x^3_{t+j} | \phi_t\}$$

As with production this leads to a linear expression for aggregate storage,

$$s_t = a_3 + b_3 \overset{*}{p}_{t+1} + b_4 p_t + c_3 x_t^3 + \varepsilon_t^3 \qquad (9)$$

with $a_3 = -\alpha_{20}/\alpha_{21}$, $b_3 = \rho/\alpha_{21}$, $b_4 = -1/\alpha_{21}$, and $c_3 = -\alpha_{22}/\alpha_{21}$. Expected storage quantities are

$$\overset{*}{s}_{t+j} = a_3 + b_3 \overset{*}{p}_{t+j+1} + b_4 \overset{*}{p}_{t+j} + c_3 x_{t+j}. \qquad (9a)$$

Equations (8) and (9) were derived without assumptions about the time paths of their arguments and hold for arbitrary assumptions about those paths so long as A2.1 - A2.5 hold. Equation (5) is assumed to possess similar invariance properties although no derivation is given in this section.[3] These behavioral equations take as given expected values. The notions of structure and historical values which imply expected values constitute the information set, Ø. We define exogenous structure as those aspects of the information set which imply expected values for the exogenous variables. Solution of the model requires specification of the information set. Changes in the information set imply changes in the solution. Changes in technology imply structural shifts as in conventional models but changes in exogenous structures or knowledge of historical values do not appear in structural shifts. The relevance of information about variable values is illustrated by noting

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[3]One of many possible derivations of a consumption demand curve with this form is given in the APPLICATION section.

that if agents know current decision quantities they can infer the values of current disturbance terms which affect the solution of the model. We have assumed that agents don't know current quantities so they cannot infer the values of these disturbances; however, knowledge of the current price allows them to infer the value of the sum $\varepsilon_t^1 + \varepsilon_t^3$.

The decision quantities can be represented in a decision vector

$$D_t = \begin{bmatrix} d_t \\ q_t \\ s_t \end{bmatrix} .$$

The state of the market each period is characterized by the quantities which will result from production and storage decisions made in previous periods. Represented as a state vector these are

$$K_t = \begin{bmatrix} s_{t-1} \\ q_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ q_{t-h} \end{bmatrix}$$

The evolution of the state vector satisfies (2) with

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ \cdot & & \\ & \cdot & \\ & & \cdot \\ 0 & 0 & 0 \end{bmatrix}$$

Market clearing imposes conditions, one for each period as given in (3) with $G = [1 \quad 0 \quad \dots \quad 0 \quad 1]$ and $H = [1 \quad 0 \quad 1]$.

Analysis

This market structure implies an equilibrium price sequence. Since the market clearing conditions and law of motion for the state vector are linear we interpret them as conditions on expected values. By successive substitution future state variables are written in terms of expected decisions and the current state; then substituting from the behavioral relations the market clearing conditions become linear equations in prices, current and expected; exogenous values, current and expected; and the current state. These equations are solved for the equilibrium price sequence. These conditions are:

for the current period

$$(b_1 + b_4)p_t + b_3\overset{*}{p}_{t+1} = \Phi_t; \tag{10}$$

for the next h-1 periods, $0 < j < h$,

$$-b_4\overset{*}{p}_{t+j-1} + (b_1 - b_3 + b_4)\overset{*}{p}_{t+j} + b_3\overset{*}{p}_{t+j+1} = \Phi_{t+j} \tag{11}$$

and for periods after that, $j \geq h$

$$-b_4 p^*_{t+j-1} + (b_1 - b_2 - b_3 + b_4) p^*_{t+j} + b_3 p^*_{t+j+1} = \Phi_{t+j} \qquad (12)$$

where

$$\Phi_t = -a_1 - a_3 - c_1 x^1_t - c_3 x^3_t - \varepsilon^1_t - \varepsilon^3_t + s_{t-1} + q_{t-h}$$

for $0 < j < h$

$$\Phi_{t+j} = -a_1 - c_1 x^{*1}_{t+j} - c_3 (x^{*3}_{t+j} - x^{*3}_{t+j-1}) + q_{t+j-h}$$

and for $j \geq h$

$$\Phi_{t+j} = a_2 - a_1 + c_2 x^{*2}_{t+j-h} - c_1 x^{*1}_{t+j} - c_3 (x^{*3}_{t+j} - x^{*3}_{t+j-1}).$$

The conditions for period t + h and beyond, equation (12), can be represented as an infinite band matrix and solved by developing the inverse [see Theil (1964), appendix 5.C, page 215]. The awkward notation and cumbersome expressions of that approach are avoided by recognizing (12) as a nonhomogeneous difference equation for which the general solution is

$$p^*_j = k_1 r_1^j + k_2 r_2^j + \Psi_j$$

where r_1 and r_2 are roots of an associated quadratic equation, k_1 and k_2 arbitrary constraints, and Ψ any particular solution of (12).

Under our assumptions the roots are real and positive for (see appendix I)

$$b_1 \leq b_2 - b_4 (\rho^{\frac{1}{2}} - 1)^2.$$

This becomes the familiar requirement that the supply schedule cut the demand schedule from below when firms maximize undiscounted profits over a finite horizon, $\rho = 1$.

Boundary conditions specify the relevant solution to (12). There is a natural stability condition which expected prices must satisfy. This is that if expected values for exogenous variables are constant then prices must be expected to converge to a constant. A solution with this property is obtained by choosing a particular solution, Ψ, with this property and making the coefficient(s) of the root(s) which are greater than unity zero. Appendix I shows that in our case at most one root will be less than or equal to unity. The additional boundary condition which determines the coefficient on this root comes from the interlocking of (10) and (11) with (12) and the presence of the current price in (10) and (11). For some values of the parameters both roots exceed unity. In such cases solutions to (12) which satisfy (11) will be unstable even when expected exogenous values are constant. The restriction on the parameters implying the existence of a root less than unity is derived in appendix I. Not surprisingly, it is

$$b_1 \leq b_2 .$$

With the larger root eliminated expected prices for $j \geq h-1$ can be written

$$\overset{*}{p}_{t+j} = kr^j + \Psi_{t+j} \qquad (13)$$

where r is the smaller root and k its coefficient. Substituting, this gives

$$\overset{*}{p}_{t+j} - rp_{t+h-1} = \Psi_{t+h} - r\Psi_{t+h-1} = \Theta_t \tag{14}$$

where, as shown in appendix I,

$$\Theta_t = -\frac{1}{b_3} \sum_{i=0}^{\infty} (r\rho)^{i+1} \Phi_{t+h+i} \cdot$$

We now have h+1 linear equations in the current price and expected prices for the next h periods. In matrix form they, (10), (11), and (14), are

$$A_o P_t = R_t \tag{15}$$

where

$$A_o = \begin{bmatrix} (b_1+b_4) & b_3 & 0 & 0 & & \\ -b_4 & (b_1-b_3+b_4) & b_3 & 0 & & \\ 0 & -b_3 & (b_1-b_3+b_4) & b_3 & & \\ & & & \cdot & & \\ & & & \cdot & & \\ & & & \cdot & & \\ & & & -b_4 & (b_1-b_3+b_4) & b_3 \\ & & & 0 & -r & 1 \end{bmatrix},$$

$$P_t = \begin{bmatrix} p_t \\ p^*_{t+1} \\ p^*_{t+2} \\ \cdot \\ \cdot \\ \cdot \\ p^*_{t+h-1} \\ p^*_{t+h} \end{bmatrix}, \quad \text{and} \quad R_t = \begin{bmatrix} \Phi_t \\ \Phi_{t+1} \\ \Phi_{t+2} \\ \cdot \\ \cdot \\ \cdot \\ \Phi_{t+h-1} \\ \Theta_t \end{bmatrix}.$$

The parameter values and structure of (15) come from technology which we assume stable, while the values of the variables in R_t come from the exogenous structure which we allow to change. The effects of changes in exogenous structure are represented as changes in the solution of the linear system in response to variations in its righthand side vector. Elements of A_o^{-1} can be interpreted as impact multipliers for changes in prices. The impacts on decisions are determined by substitution of the relevant price changes into the behavioral relations which can be written compactly as

$$Y_t = Z_t\beta + \varepsilon_t \tag{16}$$

where

$$Z_t = \begin{bmatrix} 1 & p_t & x_t^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & p^*_{t+h} & x_t^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & p^*_{t+1} & p_t & x_t^3 \end{bmatrix},$$

$$Y_t = \begin{bmatrix} d_t \\ q_t \\ s_t \end{bmatrix}, \qquad \varepsilon_t = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t \\ \varepsilon_t \end{bmatrix}, \text{ and}$$

$$\beta = (a_1 \;\; b_1 \;\; c_1 \;\; a_2 \;\; b_2 \;\; c_2 \;\; a_3 \;\; b_3 \;\; b_4 \;\; c_3).$$

We have described an approach to market analysis with rational expectations which is similar to analysis with extrapolative expectations except that the equilibrium point is a sequence which depends on beliefs about exogenous structure. While this dependence requires additional specification on the part of an analyst it also allows consideration of a wider range of questions.

III. ESTIMATION

In this section we develop estimators for the structural parameters we treated as known in the previous section. We consider the reduced form of the model to develop identification conditions. The reduced form also suggests appropriate estimators. Instrumental variables are constructed for the current and expected prices which are used to define consistent estimators for the structural parameters. Special features of the structure suggest other estimators which may provide greater asymptotic efficiency.

To consider estimation we adopt the following assumptions about stochastic structure.

A3.1 In each period endogenous values satisfy (15) and (16) where the elements of β are unknown parameters.

A3.2 The disturbance vectors, ε_t, are intertemporally independent random draws from a trivariate normal population with zero mean and an unknown nonsingular covariance matrix, Ω.

A3.3 For all t, i = 1, 2, 3, and $j \geq 1$ the variables x_t^i and x_{t+j}^{*i} are exogenous, bounded, and known.

A3.4 $b_1 \leq b_2$.

Recall that the parameters in (15) are known functions of the elements of β which appear linearly in (16). Also, by previous assumption, $b_2 > 0$, $b_3 > 0$, and $b_3 = -\rho b_4$ with $0 < \rho < 1$.

As shown in appendix I, with these conditions expected prices are dynamically stable. Actual prices are normally distributed about expected prices and the decision variables are linear functions of expected prices, the current price, and exogenous variables. Since exogenous variables are assumed bounded the system is dynamically stable.

Identification

Initially, let's consider identification for a simple special case, then see how it is affected as we generalize. Suppose that we measure all variables as deviations from their means eliminating intercepts, that the system includes no exogenous variables, and that production involves no lag. Then the structural form is

$$\begin{bmatrix} 1 & 0 & 0 & -b_1 & 0 \\ 0 & 1 & 0 & 0 & -b_2 \\ 0 & 0 & 1 & -b_4 & -b_3 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -r & 1 \end{bmatrix} \begin{bmatrix} d_t \\ q_t \\ s_t \\ p_t \\ p^*_{t+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} s_{t-1} + \begin{bmatrix} \varepsilon^1_t \\ \varepsilon^2_t \\ \varepsilon^3_t \\ 0 \\ 0 \end{bmatrix} . \qquad (17)$$

The associated reduced form is

$$\begin{bmatrix} d_t \\ q_t \\ s_t \\ p_t \\ p^*_{t+1} \end{bmatrix} = \begin{bmatrix} b_1/\Delta \\ b_2/\Delta \\ (b_4+b_3r)/\Delta \\ 1/\Delta \\ r/\Delta \end{bmatrix} s_{t+1} + \begin{bmatrix} \eta^1_t \\ \eta^2_t \\ \eta^3_t \\ \eta^4_t \\ \eta^5_t \end{bmatrix} \tag{18}$$

where $\Delta = b_1 + b_4 + (b_3 - b_2)r$ and the η^i_t are linear combinations of the three structural disturbances. Since the rank of the covariance matrix of the reduced form disturbances is only three, all information is used by estimating three equations from (18). These provide three reduced form coefficients. Each of them is a known function of the four structural parameters b_1, b_2, b_3 and b_4. In this case the structural parameters are not all identified. Notice that if we had represented agents as maximizing undiscounted profits over a finite horizon then $b_4 = -b_3$ and the structural parameters would be identified. Similarly, in the case we are now considering the parameters are identified if the discount rate is known, a priori.

Suppose that the system includes one or more exogenous variables. Each exogenous variable introduces one new structural parameter for each behavioral relation in which it appears, or at most three new structural parameters. At the same time, it provides at least three new reduced form coefficients; more than three if it has an expected value which cannot be written as a linear combination of the predetermined variables in the system. Each period of production lag introduces an additional

predetermined variable into the system, the variable q_{t-1} when $h = 1$, q_{t+1} and q_{t-2} when $h = 2$, and so on. Each of these variables provides three additional reduced form coefficients without increasing the number of structural parameters.

Thus we see that any of the following conditions are sufficient for identification.

- The discount rate, ρ, is known a priori.
- The production lag, h, is greater than zero.
- At least one of the exogenous variables, including constants, does not appear in every behavioral relation.
- At least one exogenous variable has an expected value which cannot be written as a linear combination of the predetermined variables in the system.

Consistent Estimators

A two stage instrumental variable procedure can be used to define consistent estimators of the structural parameters. Since expected prices are conditional expectations they differ from the subsequently realized actual prices by error terms having expected value zero and uncorrelated with information available at the time the expectations are held.

$$p_{t+j} = \overset{*}{p}_{t+j} + \xi_t^j$$

Therefore, we can regard actual prices as measurements, made with errors, of their unobserved prior expectations. Replacing

the expected prices in (16) by actual prices, lagged forward appropriately, transforms the unobserved variables problem into an errors in variables problem. This gives the system

$$Y_t = \ddot{Z}_t\beta + \gamma_t \qquad (16')$$

where Y_t and β are defined as in (16),

$$\ddot{Z}_t = \begin{bmatrix} 1 & p_t & x_t^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & p_{t+h} & x_t^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & p_{t+1} & p_t & x_t^3 \end{bmatrix},$$

and

$$\gamma_t' = (\varepsilon_t^1,\ \varepsilon_t^2 + b_2\xi_t^h,\ \varepsilon_t^3 + b_3\xi_t^1) .$$

Equations (16') are not appropriately estimated by ordinary least-squares because of the errors in certain variables and also because prices, both current and expected, are endogenous and thus correlated with the structural disturbances. The instrumental variable procedure deals with both of these problems. The necessary instruments are constructed by regressing the prices p_t, p_{t+1}, and p_{t+h} on a subset of the predetermined variables entering the reduced form.

Specifically consistent estimators for β require two regression stages.

Stage one. Construct instruments as follows.

$$\bar{p}_t = \chi_t[(\chi'\chi)^{-1}\chi'p]$$
$$\bar{p}_{t+1} = \chi_t[(\chi'\chi)^{-1}\chi'p(1)]$$
$$\bar{p}_{t+h} = \chi_t[(\chi'\chi)^{-1}\chi'p(h)]$$

where

$$p' = (p_1, p_2, p_3, \ldots, p_T)$$
$$p(1)' = (p_2, p_3, p_1, \ldots, p_{T+1})$$
$$p(h)' = (p_{1+h}, p_{2+h}, p_{3+h}, \ldots, p_{T+h})$$

with χ_t a row vector of values, in period t, for a subset of the predetermined variables in the system. For example,

$$\chi_t = (1, x_t^1, x_t^2, x_t^3, s_{t-1}, q_{t-1}, \ldots, q_{t+h}).$$

Stage two. Using these instruments consistent estimators for the elements of β are

$$\tilde{\beta} = (\bar{Z}' \ddot{Z})^{-1}\bar{Z}' Y \tag{19}$$

where

$$\ddot{Z} = (\ddot{Z}_1', \ddot{Z}_2', \ldots, \ddot{Z}_T'),$$
$$\bar{Z} = (\bar{Z}_1', \bar{Z}_2', \ldots, \bar{Z}_T').$$

and

$$Y' = (Y_1', Y_2', \ldots, Y_T')$$

with

$$\bar{Z}_t = \begin{bmatrix} 1 & \bar{p}_t & x_t^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \bar{p}_{t+h} & x_t^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \bar{p}_{t+1} & \bar{p}_t & x_t^3 \end{bmatrix}.$$

To establish the consistency of $\tilde{\beta}$ we substitute from (16) for Y into (19) and evaluate the probability limit. That is,

$$\text{plim } \tilde{\beta} = \text{plim}\{(\bar{Z}' \ddot{Z})^{-1}\bar{Z}' \ddot{Z} \beta\} + \text{plim}\{(\bar{Z}' \ddot{Z})^{-1}\bar{Z}' \varepsilon\}$$

which can be rewritten as

$$\text{plim } \tilde{\beta} = \beta[\text{plim } \frac{1}{T} \bar{Z}' \ddot{Z}]^{-1} [\text{plim } \frac{1}{T} \bar{Z}' Z] + [\text{plim } \frac{1}{T} \bar{Z}' \ddot{Z}]^{-1} [\text{plim } \frac{1}{T} \bar{Z}' \varepsilon].$$

Clearly, $\tilde{\beta}$ is consistent for β if

$$\text{plim } \frac{1}{T} \bar{Z}'Z \text{ is nonsingular,}$$
$$\text{plim } \frac{1}{T} \bar{Z}'\ddot{Z} = \text{plim } \frac{1}{T} \bar{Z}'Z,$$

and

$$\text{plim } \frac{1}{T} \bar{Z}'\varepsilon = 0.$$

The instruments $\bar{p}_t$, $\bar{p}_{t+1}$, and $\bar{p}_{t+h}$ are, in probablity limit, linear combinations of the elements of χ_t. The elements of χ_t are uncorrelated with current and future structural disturbances by assumption and with the measurement errors, ξ_t^j, because χ_t is included in the information available at the time expectations are formed. Since $\bar{Z}'$ consists of these instruments and exogenous variables

$$\text{plim } \frac{1}{T} \bar{Z}' \varepsilon = 0$$

follows from assumption A3.3.

From (15) we see that the prices included in Z are linearly dependent on current exogenous variables with probability zero. Therefore

$$\text{plim } \frac{1}{T} \overline{Z}' Z$$

is singular with probability zero.

Finally we observe that

$$\frac{1}{T} \overline{Z}' \ddot{Z} = \frac{1}{T} \overline{Z}' Z + \frac{1}{T} \overline{Z}' (\ddot{Z} - Z)$$

with

$$(\ddot{Z}_t - Z_t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \xi_t^h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi_t^1 & 0 & 0 \end{bmatrix} .$$

Only seven elements of $\frac{1}{T} \overline{Z}' (\ddot{Z} - Z)$ are not identically zero, they are

$$\frac{1}{T} \sum_{t=1}^{T} \xi_t^h$$

$$\frac{1}{T} \sum_{t=1}^{T} \overline{p}_{t+h} \xi_t^n$$

$$\frac{1}{T} \sum_{t=1}^{T} x_t^2 \xi_t^n$$

$$\frac{1}{T} \sum_{t=1}^{T} \xi_t^1$$

$$\frac{1}{T} \sum_{t=1}^{T} \bar{p}_{t+1} \xi_t^1$$

$$\frac{1}{T} \sum_{t=1}^{T} \bar{p}_t \xi_t^1$$

and

$$\frac{1}{T} \sum_{t=1}^{T} x_t^3 \xi_t^1 .$$

Each of these have probability limit zero because the instruments are linear combinations of predetermined variables and the stochastic terms ξ_t^i have information which includes the predetermined variables. Therefore

$$\text{plim } \frac{1}{T} \bar{Z}'(\ddot{Z} - Z) = 0$$

and

$$\text{plim } \frac{1}{T} \bar{Z}' \ddot{Z} = \text{plim } \frac{1}{T} \bar{Z}' Z.$$

This two stage procedure is similar to that suggested by McCallum (1976). The difference is the inclusion, in our case, of (15) which explicitly specifies how expected prices are determined. McCallum's approach can be regarded as a special case derived from specific assumptions about exogenous structure. His approach demonstrates a useful observation. If the exogenous structure doesn't change through the interval being estimated then we need only specify the form of the exogenous process. This form indicates the predetermined variables involved in the system, we then select χ_t from among that set.

Other Estimators

The structure suggests other estimators which may be asymptotically more efficient. Using $\tilde{\beta}$ we define estimators which are consistent for the unobserved expected prices. We then define a new estimator for β which is the two stage (or three stage if Ω is not diagonal) least-squares estimator, using the estimates of expected prices as if they were observations. Continuing iteratively, a sequence of estimators is obtained each of which is consistent for β.

The last h equations in (15) are identities which can be solved for expected prices. They can be written as

$$B_o \overset{*}{P}_t = S_t$$

where

$$B_o = \begin{bmatrix} (b_2-b_3+b_4) & b_3 & 0 & & & \\ -b_4 & (b_2-b_3+b_4) & b_3 & & & \\ & \cdot & \cdot & \cdot & & \\ & & & -b_4 & b_2-b_3+b_4 & b_3 \\ & & & 0 & -r & 1 \end{bmatrix}$$

$$\overset{*}{P}_t = \begin{bmatrix} \overset{*}{p}_{t+1} \\ \overset{*}{p}_{t+2} \\ \vdots \\ \overset{*}{p}_{t+h-1} \\ \overset{*}{p}_{t+h} \end{bmatrix}, \quad \text{and} \quad S_t = \begin{bmatrix} \Phi_{t+1}+b_4 p_t \\ \Phi_{t+2} \\ \vdots \\ \Phi_{t+h-1} \\ \Theta_t \end{bmatrix}$$

Each of the elements of B_o is a continuous function of the elements of β. Therefore the elements of B_o^{-1} are continuous functions of the elements of β which means that $\tilde{B}_o^{-1}$ is consistent for B_o^{-1} if each element of $\tilde{B}_o$ is the same function of a consistent estimator for β as the corresponding element of B_o is of β. Similarly, $\tilde{S}_t$ is consistent for S_t where each element of $\tilde{S}_t$ is the same function of a consistent estimator for β as the corresponding element of S_t is of β. This result is immediate in the case of the first h-1 elements which are simply linear functions of β. The proof for the final element is given in appendix II. Since we have $\tilde{\beta}$ which is consistent for β we define

$$\begin{bmatrix} \tilde{p}_{t+1} \\ \tilde{p}_{t+2} \\ \vdots \\ \tilde{p}_{t+h-1} \\ \tilde{p}_{t+h} \end{bmatrix} = \tilde{B}_o^{-1} \tilde{S}_t$$

which is consistent for expected prices.

We now define

$$\tilde{\beta}(n) = [\tilde{Z}'(I_3 \otimes \tilde{X})(\tilde{\Omega} \otimes \tilde{X}'\tilde{X})^{-1}(I_3 \otimes \tilde{X}')\tilde{Z}] \times \tilde{Z}'(I_3 \otimes \tilde{X})(\tilde{\Omega} \otimes \tilde{X}'\tilde{X})^{-1}(I_3 \otimes \tilde{X})Y \tag{20}$$

where $\otimes$ indicates a Kronecker product, I_3 a rank three identity matrix,

$$\tilde{X}' = (\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_T)$$

with $\tilde{X}_t$ a column vector of all the predetermined variables entering the system in period t with $\tilde{\Theta}_t$ replacing Θ_t, and

$$\tilde{Z}' = (\tilde{Z}_1', \tilde{Z}_2', \ldots, \tilde{Z}_T')$$

with

$$\tilde{Z}_t' = \begin{bmatrix} 1 & p_t & x_t^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \tilde{p}_{t+h} & x_t^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \tilde{p}_{t+1} & p_t & x_t^3 \end{bmatrix}.$$

The $\tilde{p}_{t+h}$ and $\tilde{p}_{t+1}$ are consistent estimators for p^*_{t+h} and p^*_{t+1} respectively obtained as described in the preceding paragraph using $\tilde{\beta}(n-1)$.

Substituting from (16) for Y in (20) we see that $\tilde{\beta}(n)$ is consistent for β if

$$\text{plim } \frac{1}{T}(I_3 \otimes \tilde{X}')\tilde{Z} = \text{plim } \frac{1}{T}(I_3 \otimes \tilde{X}')Z = \text{plim } \frac{1}{T}(I_3 \otimes X')Z$$

and

$$\text{plim } \frac{1}{T} (I_3 \otimes \tilde{X}')\varepsilon = \text{plim } \frac{1}{T} (I_3 \otimes X')\varepsilon = 0$$

These conditions hold as special cases of the law of large numbers [see Gnedenko (1968), page 25].[4] By the same reasoning

$$\tilde{\Omega} = \frac{1}{T} \sum_{t=1}^{T} (Y_t - \tilde{Z}_t\tilde{\beta})(Y_t - \tilde{Z}_t\beta)'$$

is consistent for Ω.

For certain assumptions about the exogenous structure the infinite sum we have represented as Θ_t can be replaced by a linear function of a small number of variables and coefficients which are known functions of the basic parameters. For example, if each of our three exogenous variables follow random walk processes

$$\Theta_t = \frac{(a_2-a_1)r}{1-r} - \frac{c_1 r}{1-r} x_t^1 + \frac{c_2 r}{1-r} x_t^2 .$$

In such cases the reduced form coefficients are known [nonlinear] functions of the structural parameters. Malinvauld (1966), chapter nine, section five describes asymptotically efficient estimators for such systems.

This replacement of Θ_t is possible with exogenous structures such that the method of undetermined coefficients can be used to

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[4] If

$$\text{plim } \tilde{x}_t^1 = x_t^1 \text{ and plim } \tilde{x}_t^2 = x_t^2,$$

and

$$\text{plim } \frac{1}{T} \sum_{t=1}^{T} \tilde{x}_t^1 \tilde{x}_t^2 = \text{plim } \frac{1}{T} \sum_{t=1}^{T} x_t^1 x_t^2 .$$

find a particular solution, Ψ, to (12). This includes all structures in which exogenous variables are generated by finite order autoregressive processes.

IV. APPLICATION

This section presents a rational expectations model for the United States broiler chicken market. Behavioral relations are developed from agents' decision problems. Results from estimation of this model for three years of weekly data are reported.

The Broiler Market

In usage broilers compete most closely with the fresh meats, beef and pork. In recent years annual per capita consumption of these products has been fairly stable at about 115 pounds of beef, 60 pounds of pork, and 37 pounds of broiler chicken. After slaughter virtually all broilers are "chilled" and marketed as fresh, primarily to retail food dealers. A small part of the slaughter is sold to manufacturers of frozen dinners, fast food outlets, or frozen for export.

To develop a usage equation we characterize the representative user as choosing quantities of beef, pork, and broilers to maximize their profit from the resale of those products. The users are regarded as individually small relative to the wholesale markets -- regarding their purchase prices as given. They are not small relative to the markets in which they sell but take demands for their retail products as linear in the retail prices. Specifically,

$$\begin{bmatrix} dbf_t \\ dpk_t \\ dch_t \end{bmatrix} = 2\ G \begin{bmatrix} rpbf_t \\ rppk_t \\ rpch_t \end{bmatrix} + \begin{bmatrix} W_t^1 \\ W_t^2 \\ W_t^3 \end{bmatrix} \tag{21}$$

where G is a three-dimensional nonsingular matrix, dbf_t, dpk_t, and dch_t are quantities and $rpbf_t$, $rppk_t$, and $rpch_t$ retail prices, respectively, of beef, pork, and broilers. The W_t^i terms represent other influences shifting the demands for these goods. The optimal purchase for such a firm is the solution of

$$\text{Max} \begin{bmatrix} dbf_t, & dpk_t, & dch_t \end{bmatrix} \left(\begin{bmatrix} rpbf_t \\ rppk_t \\ rpch_t \end{bmatrix} - \begin{bmatrix} pbf_t \\ ppk_t \\ pch_t \end{bmatrix} \right) \tag{22}$$

subject to (21), where pbf_t, ppk_t, and pch_t are respectively the wholesale prices of beef, pork, and broilers. Solving (21) for retail prices in terms of quantities and substituting into (22) gives an unconstrained quadratic programming problem. With G negative definite the solution is

$$\begin{bmatrix} dbf_t \\ dpk_t \\ dch_t \end{bmatrix} = G \begin{bmatrix} pbf_t \\ ppk_t \\ pch_t \end{bmatrix} + \begin{bmatrix} W_t^1 \\ W_t^2 \\ W_t^3 \end{bmatrix} .$$

Consumption of broilers is consistently higher in the summer. This seasonal variation in demand is represented by a periodic cubic spline with knots in the third, twenty-second, and thirty-sixth weeks. The major holidays cause substantial

decreases in slaughter. These effects are represented by a dummy variable, D_t, which takes a value of unity in those weeks containing Christmas Day, New Year's Day, Memorial Day, Independence Day, Labor Day, Thanksgiving Day, and the week following Thanksgiving; otherwise its value is zero. Assuming that all other influences are adequately represented by a constant and normally distributed error the usage equation for broilers is

$$dch_t = a_{11}SS1_t + a_{12}SS2_t + a_{13}SS3_t + b_1 pch_t + c_{11} pbf_t + c_{12} ppk_t + c_{13} D_t + \varepsilon_t^1 . \quad (23)$$

We expect

$$b_1 < 0,\ c_{11} > 0,\ c_{12} > 0,\ c_{13} < 0.$$

Broilers are a hybrid strain developed for rapid weight gain and efficient conversion of feed into high quality meat. Almost all commercially marketed broilers in this country are produced by vertically integrated firms. Such firms typically operate a feedmill, processing plant, and hatchery, and maintain a breeder flock. They usually contract with individual farmers to grow-out the chicks. The integrated producer maintains ownership of the birds and provides the contractor with feed, medical supplies, etc. The birds are fed a carefully formulated ration consisting of about 60 percent yellow corn and 20 percent soybean meal with the remaining 20 percent drawn from a variety of sources to provide a proper balance of nutrients [see Parkhurst (1967)].

The birds reach slaughter weight in slightly less than eight weeks. They are then collected for "processing" which is a mechanized factory type operation. Producers have made some attempts to promote frozen broilers but have been unable to overcome users' resistance. This resistance precludes storage of slaughtered birds; however, it is possible to delay slaughter thus storing live birds. They are stored in this manner for only a few days because their rate of conversion of feed to meat declines, they become too large for the processing equipment, and users will not accept "oversize" birds.

To develop behavioral relations for production and storage we assume that production and storage costs can be represented separately as follows. For production in period t

$$C_t^q(qch) = [\alpha_1 pcf_t + \alpha_2 qch]\ qch$$

and for storage in that period

$$C_t^s(sch) = [\alpha_3 pcf_t + \alpha_4 sch]\ sch\ ,$$

where qch, sch, and pcf are, respectively, the number of chicks placed, the number of birds "stored," and the price of feed. Optimal placement and storage quantities are solutions to

$$\text{Max} \sum_{j=0}^{\infty} \rho^j \{\overset{*}{p}_{t+j}\ (ws_{t+j} + qch_{t+j-h} - sch_{t+j})$$
$$- [\alpha_1 \overset{*}{pcf}_{t+j} + \alpha_2 qch_{t+j}]\ qch_{t+j}$$
$$- [\alpha_3 \overset{*}{pcf}_{t+j} + \alpha_4 sch_{t+j}]\ sch_{t+j}\}$$

where w is the ratio of the weight of stored birds to those not stored and α_2 and α_4 are assumed positive.

Adding constants and normally distributed error terms to represent omitted influences this characterization of the production process gives the following behavioral relations.

For production,

$$qch_t = a_2 + b_2 \overset{*}{p}ch_{t+8} + c_2 pcf_t + \varepsilon_t^2 \qquad (24)$$

and for storage,

$$sch_t = a_3 + b_3 \overset{*}{p}ch_{t+1} + b_4 pch_t + c_3 pcf_t + \varepsilon_t^2 \; . \qquad (25)$$

Since

$$b_2 = \frac{\rho^8}{2\alpha_2} \; , \quad b_3 = \frac{\rho w}{2\alpha_4} \; , \quad \text{and } b_4 = -\frac{1}{2\alpha_4}$$

we expect

$$b_2 < 0, \; b_3 < 0, \text{ and } b_4 < 0 \; ,$$

with b_3 either smaller or larger than the absolute value of b_4 depending on whether ρw is less than or greater than unity. Similarly we expect

$$c_2 < 0, \text{ and } c_3 < 0 \; .$$

Data

Weekly data for 1973, 1974, and 1975 was used to estimate equations (23), (24), and (25). Quantity figures used came from USDA reports, nominal prices from the Wall Street Journal,

and a wholesale price index for all commodities from the Survey of Current business. We used the number of broiler chickens placed in important broiler producing states as the quantity started into production. These numbers are reported for weeks ending in Saturday each year in a USDA report entitled Commercial Broilers. We used the federally inspected slaughter as the quantity demanded. Both the number of birds slaughtered and their average weight for each week ending in Wednesday are reported in the USDA annual publication, Poultry Market Statistics. These series were temporally aligned with the slaughter week ending three days earlier than the placement week. This makes birds placed at the end of a week 53 days old at the end of the slaughter week eight weeks later. This is closer to the average grow out period than either 49 or 56 days. Since movement to market takes a few days, which is the reason the slaughter week ends on Wednesday, we used spot prices reported for Wednesdays. They are the cash prices for broilers in New York, beef and pork loins in the Midwest, corn in Chicago and soybean meal in Decator, Illinois. The corn price was in units of dollars per bushel and the soybean meal price in dollars per ton. A nominal price of feed was calculated as

$$(2000/56) \times (0.775)\ pcn_t + (0.225)\ psb_t$$

where pcn and psb are the nominal prices of corn and soybean

meal, respectively.[5] The relative prices used in estimation were obtained by dividing the nominal price by the wholesale price index for the month. The prices of the nearest futures contracts for beef, hogs, corn, and soybean meal, divided by the wholesale price index, were taken as expected prices for these commodities.

The number of birds stored was calculated using the following equation.

$$sch_t = R_s \, qch_{t-8} - nch_t + sch_{t-1}$$

with R_s the average survival rate over the three-year period and nch_t the number of birds slaughtered. If the survival rate over the period is constant such a series differs from actual quantities by a constant which is collapsed into the intercept terms. We set the initial storage quantity at zero and approximated the survival rate with

$$R_s = \sum_{t=9}^{T+8} nch_t \; / \; \sum_{t=1}^{T} qch_t \; .$$

Results

Instruments were constructed for pch_t, $\overset{*}{pch}_{t+1}$, and $\overset{*}{pch}_{t+8}$ by regressing the prices for broilers on a subset of the predetermined variables in the reduced form equations for prices. The regressors were the current and near futures prices for

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[5]Corn weighs 56 pounds per bushel.

beef, pork, and feed; the numbers of chicks placed in each of the previous eight weeks; the calculated storage from the previous week; the dummy variable for holiday weeks; the three periodic spline (seasonal) variables; and the dummy variable and two of the spline variables for the next eight weeks. The regressands were current prices for broilers, pch_t; prices for broilers lagged forward one period, $pch(1)_t$; and the prices for broilers lagged forward eight periods, $pch(8)_t$. The simple correlation coefficients between the regressands and the predicted values in each regression are given below.

Variables	Simple correlation coefficients
pch_t vs. $\bar{p}ch_{t+1}$	0.9072
$pch(1)_t$ vs. $\bar{p}ch(1)_t$	0.8995
$pch(8)_t$ vs. $\bar{p}ch(8)_t$	0.9221

The results for the instrumental variable regressions are given below. Standard errors for the coefficient estimates are given in parenthesis below the estimated coefficients. The units of measurement for the variables are indicated below the variables. Following each equation is the standard error for the equation and simple correlation coefficients for the residuals.[6]

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[6]For each equation the sum of squared errors divided by the degrees of freedom is reported as an estimate of the variance of the equation error. This somewhat overstates the variance in the cases of the production and storage equations because of the errors in the expected price variables. Since the coefficients of interest in these relations are already strongly significant no adjustment was made.

Demand

$$dch_t = 229.1\ SS1_t + 206.8\ SS2_t + 228.4\ SS3_t$$

millions of lbs. (7.627) (6.853) (7.523)

$$- 37.77\ D_t + 1.276\ pbf_t + 0.6574\ ppk_t$$

(3.120) (0.3900) \$/cwt. (0.4120) \$/cwt.

$$- 2.182\ pch_t$$

(0.5438) \$/cwt.

Equation standard error 12.55

Simple correlation coefficients for residuals

t vs. t-1	0.1919
t vs. t-2	-0.0269
t vs. t-3	0.0408
t vs. t-4	0.1013
t vs. t-5	0.0624
t vs. t-6	0.2435
t vs. t-7	0.0374
t vs. t-8	0.0050
t vs. t-9	0.1124
t vs. t-10	0.0417

Production

$$qch_t = 52.55 - 0.09855\ pcf_t + 0.4387\ \overset{*}{pch}_{t+8}$$

millions of birds (2.356) (0.03217) \$/ton (0.08896) \$/cwt.

Equation standard error 4.270

Simple correlation coefficients for residuals

t vs. t-1	0.7996
t vs. t-2	0.6241
t vs. t-3	0.5777
t vs. t-4	0.6080
t vs. t-5	0.5614
t vs. t-6	0.4756
t vs. t-7	0.4130
t vs. t-8	0.3661
t vs. t-9	0.2860
t vs. t-10	0.2285

Storage

sch_t = 1.131 + 0.9738 pcf_t - 4.444 $\overset{*}{pch}_{t+1}$

millions of birds (9.061) (0.1225) $/ton (1.145) $/cwt.

+ 2.939 pch_t

(1.103) $/cwt.

Equation standard error 16.05

Simple correlation coefficients for residuals

t vs. t-1	0.7012
t vs. t-2	0.5788
t vs. t-3	0.5726
t vs. t-4	0.4688
t vs. t-5	0.4312
t vs. t-6	0.4180

t vs. t-7	0.3479
t vs. t-8	0.2811
t vs. t-9	0.2228
t vs. t-10	0.1614

With this sample size, 152 observations, all of these coefficients, except that for the price of pork in the demand relation and the constant in the storage equation, are significant at the 0.99 level. The coefficient for the price of pork in the demand equation is significant at the 0.90 level. The coefficients in the demand and production equations have the signs indicated by our hypothesis. The coefficients in the storage equation are contrary to those indicated by our hypothesis.

Two alternative specifications of the market process were investigated. The first linked production and storage costs and the second assumed that producers don't attempt to increase profits by "storing" live birds.

In the first case the cost function for the producers was generalized by adding terms which were quadratic in slaughter and in the total grow out flock. This has the effect of introducing additional variables into the production and storage equations. Estimation of the implied storage equation produced unsatisfactory results. Although the equation standard error was considerably reduced many of the coefficients, including those for $\overset{*}{pch}_{t+1}$ and pch_t, while significant had contrary signs.

In the second alternative the difference between slaughter and production started eight weeks previously is assumed to result from unplanned occurances rather than attempts to increase profits. This implies a simpler model of the market process. The demand and production equations are unchanged and the market clearing condition becomes

$$dch_t = w(8) \cdot R_s \cdot qch_{t-8} + u_t \qquad (26)$$

with w(8) representing the weight of birds at eight weeks and u_t a stochastic term representing the deviations from scheduled slaughter. To solve for an expected price we substitute from the behavioral relation into (26). Instruments are then constructed and the structural equations estimated as before. In this case the coefficient estimates for the reduced form equations are of interest because if u_t is assumed to be independently distributed with zero mean then the reduced form coefficients are asymptotically efficient estimators of known functions of the structural parameters.

The estimation results for the reduced form equations for the current price and the expected price for eight weeks ahead are presented below.

Reduced form for pch_t

$$pch_t = -(1/b_1)\ [a_{11}SS1_t + a_{12}SS2_t + a_{13}SS3_t + c_{11}pbf_t$$

$$+ c_{12}ppk_t + c_{13}D_t - R_s q_{t-8} + \varepsilon_t^1 - u_t]$$

Estimation results

$$pch_t = \underset{(4.761)}{14.93}\ SS1_t + \underset{(4.348)}{13.28}\ SS2_t + \underset{(5.061)}{17.01}\ SS3_t - \underset{(0.7137)}{0.05560}\ D_t$$

$$+ \underset{(0.07263)}{0.4848}\ pbf_t + \underset{(0.06742)}{0.5127}\ ppk_t - \underset{(0.08727)}{0.2625}\ qch_{t-8}$$

Simple correlation coefficients for residuals

t vs. t-1	0.6305
t vs. t-2	0.3738
t vs. t-3	0.4014
t vs. t-4	0.3892
t vs. t-5	0.2859
t vs. t-6	0.2305
t vs. t-7	0.2167
t vs. t-8	0.1691
t vs. t-9	0.0948
t vs. t-10	0.0018

Reduced form for $\overset{*}{pch}_{t+8}$

$$\overset{*}{pch}_{t+8} = -[1/(b_1 - R_s \cdot b_2)][-R_s \cdot a_2 + a_{11}\overset{*}{SS1}_{t+8} + a_{12}\overset{*}{SS2}_{t+8}$$

$$+ a_{13}\overset{*}{SS3}_t + c_{11}\overset{*}{pbf}_{t+8} + c_{12}\overset{*}{ppk}_{t+8} + c_{13}\overset{*}{D}_{t+8}$$

$$- R_s \cdot c_2 pcf_t + \overset{*}{\varepsilon}^i_{t+8} - R_s \varepsilon^2_t - \overset{*}{u}_{t+8}]$$

Estimation results

$$\overset{*}{pch}_{t+8} = \underset{(2.485)}{0.6069}\ \overset{*}{SS1}_{t+8} - \underset{(2.443)}{2.077}\ \overset{*}{SS2}_{t+8} + \underset{(2.220)}{2.898}\ \overset{*}{SS2}_{t+8}$$

$$+ \underset{(0.8238)}{0.8134}\ \overset{*}{D}_{t+8} + \underset{(0.09182)}{0.2429}\ \overset{*}{pbf}_{t+8} + \underset{(0.08968)}{0.3896}\ \overset{*}{ppk}_{t+8}$$

$$+ \underset{(0.02812)}{0.1285}\ pcf_t$$

Simple correlation coefficients for residuals

t vs. t-1	0.7704
t vs. t-2	0.5571
t vs. t-3	0.5091
t vs. t-4	0.4730
t vs. t-5	0.4077
t vs. t-6	0.3146
t vs. t-7	0.2841
t vs. t-8	0.2757
t vs. t-9	0.2349
t vs. t-10	0.2118

The results for the instrumental variable regressions of the structural equations are as follows.

Demand

$$dch_t = \underset{(16.15)}{239.1\ SS1_t} + \underset{(14.27)}{213.5\ SS2_t} + \underset{(17.05)}{245.4\ SS3_t} - \underset{(6.538)}{34.32\ D_t}$$

$$+ \underset{(1.447)}{4.045\ pbf_t} + \underset{(1.681)}{3.994\ ppk_t} - \underset{(3.065)}{8.730\ pch_t}$$

Equation standard error 25.59

Simple correlation coefficients for residuals

t vs. t-1	0.4506
t vs. t-2	0.1849
t vs. t-3	0.2544
t vs. t-4	0.2922
t vs. t-5	0.2155
t vs. t-6	0.2549
t vs. t-7	0.1267
t vs. t-8	0.0919
t vs. t-9	0.0878
t vs. t-10	-0.0142

Production

$$qch_t = \underset{(3.151)}{45.16} - \underset{(0.04369)}{0.2073\ pcf_t} + \underset{(0.1504)}{1.000\ \overset{*}{pch}_{t+8}}$$

Equation standard error 5.120

Simple correlation coefficients for residuals

t vs. t-1	0.7783
t vs. t-2	0.5612
t vs. t-3	0.5173
t vs. t-4	0.5632
t vs. t-5	0.5292
t vs. t-6	0.4589
t vs. t-7	0.4219
t vs. t-8	0.3926
t vs. t-9	0.3147
t vs. t-10	0.2682

The second stage estimates of the structural parameters all have the signs indicated by our hypothesis and are significant at the 0.99 level. The results for the reduced forms are generally consistent but weaker. This is not surprising since there is no reason to suppose that the stochastic terms u_t are independent through time. Autocorrelation in the u_t's makes our ordinary least squares estimators of the reduced form coefficients inconsistent; however, our two stage estimators of the structural parameters are still consistent.

V. SUMMARY

We developed this analysis using the rational expectations hypothesis in the context of a structural form model. This approach maintains the familiar invariance characteristics of extrapolative expectations models while providing the internal consistency of rational expectations. The dependence of expectations and hence of behavior on perceptions of exogenous structure are explicit. The benefits resulting from careful consideration of these economically relevant aspects of the process should compensate for the additional specification required.

The approach can be summarized as follows. From assumptions about technology come structural equations describing behavior conditional on expectations about future values. With certainty equivalence expectations are fully represented as means of predictive probability distributions. The structure is solved for the fixed point sequence of prices in terms of the structural parameters and expected exogenous values. This is crucial to the procedure. The reduced form expressions for prices, current and expected, provide the framework for analysis and also for the development of estimators for the structural parameters. We developed the solution procedure for a class of models in which speculative inventories are held, and indicated the solution for the case when speculative

inventories are not held. The approach seems more general, an extension of particular interest would be to develop the solutions for models with lagged decisions involved in the behavioral relations.

This approach suggests other directions for further work. The framework provides consistent estimators for expected prices in each period. Since expectations play an important role in many models such estimators will be useful. Additionally, they can be compared with futures prices and reported expectations data. As indicated in section III these estimators for expected prices suggest an iterative procedure which may provide estimators with greater asymptotic efficiency. Finally, we observe that an approach has been developed which can be applied with minor modifications to a wide class of actual markets. Such applications should contribute to a better understanding and description of those market processes.

APPENDIX I

The characteristic equation associated with (12) is

$$b_3 r^2 + (b_1 - b_2 - b_3 + b_4)r - b_4 = 0$$

Its roots are

$$\frac{-(b_1 - b_2 - b_3 + b_4) \pm [(b_1 - b_2 - b_3 + b_4)^2 + 4b_3 b_4]^{\frac{1}{2}}}{2b_3}$$

The product of these roots is

$$r_1 r_2 = -b_4 b_3^{-1} = \rho^{-1}.$$

The product of the two roots is positive so they have the same sign. Thus the conditions for positive real roots are

$$b_1 - b_2 - b_3 + b_4 < 0$$

and

$$[(b_1 - b_2 - b_3 + b_4)^2 + 4b_3 b_4] \geq 0.$$

Using $b_3 = -\rho b_4 < 0$ these can be written

$$b_1 < b_2 - b_4(\rho + 1)$$

and

$$b_1 \leq b_2 - b_4 (1 - \rho^{\frac{1}{2}})^2.$$

Since $0 < (1 - \rho^{\frac{1}{2}})^2 < 1 < (\rho + 1)$

The second condition is more restrictive.

Representing the two roots as r_1 and r_2 we show that at most one of them is less than or equal to unity. We have

$$r_1 r_2 = \rho^{-1}.$$

Suppose that

$$0 < r_1 \leq 1$$

then

$$0 < \rho r_1 < 1$$

and

$$r_2 = (\rho r_1)^{-1} > 1.$$

The smaller root is less than or equal to unity if

$$-[(b_1 - b_2 - b_3 + b_4)^2 + 4b_3 b_4]^{\frac{1}{2}} \leq b_1 + b_3 + b_4 - b_2 .$$

Now

$$1 < \rho < \rho^{\frac{1}{2}} < 0$$

$$1 > 1 - \rho > 1 - \rho^{\frac{1}{2}} > (1 - \rho^{\frac{1}{2}})^2 > 0$$

and

$$b_4 < 0$$

so

$$b_1 + b_3 + b_4 - b_2 = b_1 - b_2 + b_4(1 - \rho)$$

$$< b_1 - b_2 + b_4(1 - \rho^{\frac{1}{2}})^2 \leq 0.$$

Therefore the smaller root is less than or equal to unity when

$$(b_1 - b_2 - b_3 + b_4)^2 + 4b_3b_4 \geq (b_1 + b_3 + b_4 - b_2)^2$$

$$(b_1 - b_2)^2 + 2(b_1 - b_2)(b_4 - b_3) + (b_4 - b_3)^2$$

$$\geq (b_1 - b_2)^2 + 2(b_1 - b_2)(b_3 + b_4) + (B_3 + b_4)^2$$

$$2(b_1 - b_2)(b_4 - b_3) \geq 2(b_1 - b_2)(b_3 + b_4) .$$

Since

$$b_4 - b_3 = b_4(1 + \rho) - b_4(1 - \rho) = b_4 + b_3$$

this condition holds for

$$b_1 \leq b_2 .$$

Equation (12) can be written as

$$p_{t+j+1} - (r_1+r_2)p_{t+j} + r_1r_2p_{t+j-1} = \frac{1}{b_3} \Phi_{t+j}. \qquad (12')$$

A particular solution with the necessary stability properties is generated by reduction of order. This solution is

$$\Psi_{t+j} = - \frac{1}{b_3} \sum_{i=0}^{\infty} r_2^{-(i+1)} \sum_{k=0}^{i+j} r_1^k \Phi_{t+i+j-k}$$

with r_1 the root less than or equal to unity. We can verify that it is a solution by substituting into (12'). Since

$$r_2 > 0$$

this solution is bounded if the Φ are bounded and constant if the Φ are constant.

The variable θ_t has a slightly simpler form. Evaluating:

$$\theta_t = \psi_{t+h} - r_1 \psi_{t+h-1}$$

$$\theta_t = -\frac{1}{b_3} \sum_{i=0}^{\infty} r_2^{-(i+1)} \sum_{k=0}^{i+h} r_1^k \phi_{t+i+h-k} - r_1 \sum_{i=0}^{\infty} r_2^{-(i+1)}$$

$$\sum_{k=0}^{i+h-1} r_1^k \phi_{t+i+h-1-k}$$

$$\theta_t = -\frac{1}{b_3} \sum_{i=0}^{\infty} r_2^{-(i+1)} \sum_{k=0}^{i+h} r_1^k \phi_{t+i+h-k}$$

$$- \sum_{k=0}^{i+h-1} r_1^{k+1} \phi_{t+i+h-1-k}$$

Since

$$\sum_{k=0}^{i+h} r_1^k \phi_{t+i+h-k} = \phi_{t+i+h} + r_1 \phi_{t+i+h-1} + \cdots r_1^{i+h-1} \phi_{t+1}$$

$$+ r_1^{i+h} \phi_t$$

and

$$\sum_{k=0}^{i+h-1} r_1^{k+1} \phi_{t+i+h-1-k} = r_1 \phi_{t+i+h-1} + r_1^2 \phi_{t+i+h-2}$$

$$+ \cdots r_1^{i+h-1} \phi_{t+1} + r_1^{i+h} \phi_t ,$$

canceling like terms gives

$$\theta_t = -\frac{1}{b_3} \sum_{i=0}^{\infty} r_2^{-(i+1)} \phi_{t+i+h}$$

or with the smaller root being r

$$\theta_t = -\frac{1}{b_3} \sum_{i=0}^{\infty} (r\rho)^{i+1} \Phi_{t+i+h} \cdot$$

APPENDIX II

For fixed exogenous structure we show that

$$\theta_t = - \sum_{i=0}^{\infty} (r\rho)^{i+1} \left[\frac{1}{b_3} \Phi_{t+h+i}\right] , \qquad \text{(i)}$$

with $\rho = -b_3/b_4$, is a continuous function of the elements of in the region

$$b_2 > 0,\ b_1 \leq b_2,\ b_4 < 0,\ 0 < b_3 < -b_4 .$$

Since the probability limit of a continuous function is that function of the probability limits of its arguments [see Theil (1971), proposition (iv), page 371]. This establishes

$$\text{plim } \tilde{\theta}_t = \theta_t .$$

Recall that Appendix I showed that in this region

$$0 < r \leq 1 .$$

Since

$$0 < \rho < 1$$

we can define

$$\underline{r} = r\rho$$

and observe that

$$0 < \underline{r} < 1 .$$

Also, recall

$$\Phi_{t+h+i} = a_2 - a_1 - c_1 x^{*1}_{t+h+i} + c_2 x^{*2}_{t+i} - c_3 (x^{*3}_{t+h+i} - x^{*3}_{t+h+i-1}) .$$

With expected exogenous values fixed the terms in (i) enclosed in squared brackets are linear functions of ratios of elements of β and therefore are continuous. Since the exogenous values are bounded by assumption, so are the expressions in square brackets for every finite β in this region,

$$\left|\frac{1}{b_3} \Phi_{t+h+i}\right| < M ,$$

using the vertical lines to indicate the absolute value. Then

$$|\theta_t| < M \sum_{i=0}^{\infty} \underline{r}^{i+1} .$$

The righthand side of this expression is a power series which is absolutely convergent in the region we are considering. It is therefore uniformly convergent in this region [see Abel's second theorem, Smirnov (1964), page 388]. It follows that the series θ_t is uniformly convergent (Weierstrass's test). If the terms of a series are continuous in a region and the series is uniformly convergent, its sum is also continuous in that region [see Smirnov (1964), page 383]. Since the terms in square brackets are continuous functions of the elements of β and so are the terms $\underline{r}^{i+1}$ we have the desired result; that θ_t is a continuous function of the elements of β.

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